

# Virtual D-Branes

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Using a formalism developed to include collective coordinates, we calculate the contributions to S-matrix elements due to off-shell D-branes in a string coupling expansion. The formalism is further used to establish a two-dimensional computation of higher order corrections to the D-brane tension, both for the bosonic and the supersymmetric cases.

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## 1. Introduction

In this paper, we apply the collective coordinate formalism developed in an earlier work [1] to include higher order corrections for scattering off D-branes [2]. One goal is to explicitly show how off-shell D-branes appear as intermediate states in such amplitudes. This constitutes a check that the formalism properly accounts for the brane's recoil at the quantum mechanical level. One does indeed expect that virtually recoiling D-branes should be properly included in the formalism. After all, the translational zero modes to which these collective coordinates are related already appear in the world-sheet formulation as fully quantized massless open string states.

In the first part of the paper, we show how the formalism includes the contribution from off-shell recoiling branes by considering the scattering of a closed string state off a Dirichlet 0-brane. In the last part, we establish a formula for the corrections to the D-brane tension [3] and calculate them in the bosonic case using a conformal field theory approach. This two-dimensional formula is then used to illustrate, to this order, the expected non-renormalization of the mass of BPS saturated branes in the supersymmetric case.

## 2. Virtual Recoil

For the purpose of illustrating what to expect from virtually recoiling heavy branes, we first consider the simple case of a light scalar field (representing a light closed string state, for example the closed string tachyon) scattering off a heavy particle (representing the 0-brane). The coupling of the light particle to the heavy one,  $\lambda$ , is taken to be independent of the string coupling constant. This mimics faithfully the coupling of the closed string tachyon to the D-brane. This coupling can be calculated, at tree level, as the one-point closed string tachyon amplitude on the disc, thereby obtaining

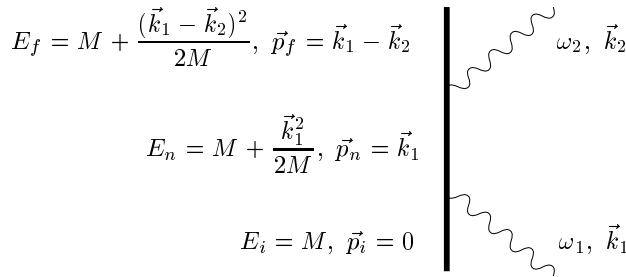
$$\lambda_0 = \frac{1}{2\pi}.$$

To evaluate the scattering amplitude of a heavy particle by a light one, it is helpful to use “old-fashioned” perturbation theory. The initial state for the process consists of a heavy particle (of mass  $M$ ) at rest ( $\vec{p} = 0$ ) and a light particle with energy  $\omega_1$  and momentum  $\vec{k}_1$ . The final state has a light particle with energy  $\omega_2$  and momentum  $\vec{k}_2$  and a heavy particle with momentum  $\vec{k}_1 - \vec{k}_2$  and energy  $M + (\vec{k}_1 - \vec{k}_2)^2/2M$ . At each

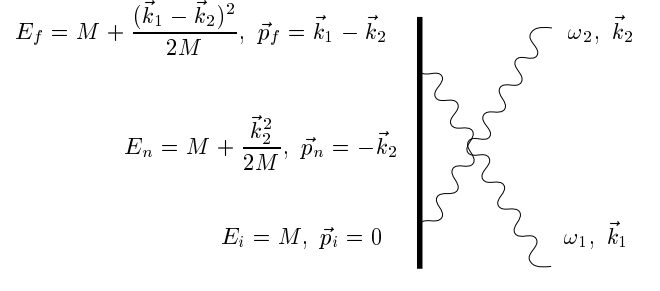
interaction vertex, momentum is conserved, whereas energy is only conserved between the initial and final states. This implies

$$M + \omega_1 = M + \frac{(\vec{k}_1 - \vec{k}_2)^2}{2M} + \omega_2.$$

Since our purpose is to focus on the contribution from virtually recoiling branes, we need to consider, in this example, two diagrams<sup>†</sup> at the lowest relevant order in perturbation theory, shown in Fig. 1 and Fig. 2. The first diagram contains a one particle intermediate state: a heavy particle with energy  $M + \vec{k}_1^2/2M$  and momentum  $\vec{k}_1$ . The second diagram contains a three particle intermediate state: a heavy particle with energy  $M + \vec{k}_2^2/2M$  and momentum  $-\vec{k}_2$  and two light particles with energies  $\omega_1, \omega_2$  and momenta  $\vec{k}_1, \vec{k}_2$ , respectively.



**Fig. 1:** The diagram with a one particle intermediate state.



**Fig. 2:** The diagram with a three particle intermediate state.

The amplitude corresponding to Fig. 1 is

$$A_1 = \frac{\lambda^2}{(M + \omega_1) - (M + \frac{\vec{k}_1^2}{2M})} = \frac{\lambda^2}{\omega_1 - \frac{\vec{k}_1^2}{2M}},$$

while that for Fig. 2 is

$$A_2 = \frac{\lambda^2}{(M + \omega_1) - (M + \frac{\vec{k}_2^2}{2M} + \omega_1 + \omega_2)} = \frac{-\lambda^2}{\omega_2 + \frac{\vec{k}_2^2}{2M}}.$$

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<sup>†</sup> There are also contributions from contact interactions, but, to this order in string coupling, the quartic interaction need only be used to first order in perturbation theory.

Combining these contributions to the scattering amplitude that involve a virtually recoiling particle gives

$$A = \frac{\lambda^2}{M} \frac{\vec{k}_1 \cdot \vec{k}_2}{\omega_1^2} \left( 1 - \frac{\vec{k}_1 \cdot (\vec{k}_1 - \vec{k}_2)}{M\omega_1} \right)^{-1}. \quad (2.1)$$

The leading term, since  $M$  is taken to be of order  $g^{-1}$ , is

$$A_0 = \frac{\lambda^2}{M} \frac{\vec{k}_1 \cdot \vec{k}_2}{\omega_1^2}. \quad (2.2)$$

This expression agrees, to the same order in  $g$ , with the result obtained in [1] for the S-matrix element associated to a closed string tachyon scattering off a 0-brane when factorizing onto intermediate translational zero-modes. It is then simple to read off the D-brane's mass,  $M$ , from (2.2) and find that it agrees with that obtained previously in [3,1]. This way of identifying the mass is well suited for calculating higher order corrections to the brane's mass, as is shown in section 3.

The contributions arising from the heavy particle being off-shell first appear in the subleading terms of equation (2.1). We now show how the formalism developed in [1] reproduces these terms. As was explained in [1], conformal invariance requires, in particular, the presence in the two dimensional action of the coupling<sup>‡</sup>

$$\frac{1}{8\pi} \oint ds \vec{a}(X_0) \cdot \partial_n \vec{X}, \quad (2.3)$$

with  $X_0$  being target time and

$$\vec{a}(X_0) = \frac{(\vec{k}_1 - \vec{k}_2)X_0}{M}.$$

The inclusion of this term in the action modifies the amplitude for the scattering of a closed string tachyon off the D-brane

$$S = \left\langle g \int d^2 z_1 e^{ik_1 \cdot X(z_1)} g \int d^2 z_2 e^{-ik_2 \cdot X(z_2)} \right\rangle_{\text{Disc}}. \quad (2.4)$$

This amplitude factorizes onto intermediate open string states in the limit where one of the vertex operators approaches the boundary. Including the new term (2.3), we obtain in this limit

$$S = g \int_0^1 r dr r^{2(\omega_1 \omega_2 - \vec{k}_1 \cdot \vec{k}_2)} (1 - r^2)^{-2-2\omega_1^2} (1 - r^2)^{2\omega_1 \vec{k}_1 \cdot (\vec{k}_1 - \vec{k}_2)/M}. \quad (2.5)$$

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<sup>‡</sup> Note that this is a logarithmic operator [4].

From (2.5), one observes that all open string poles are shifted from their lowest order values. In particular, the contribution from the translational zero mode is modified

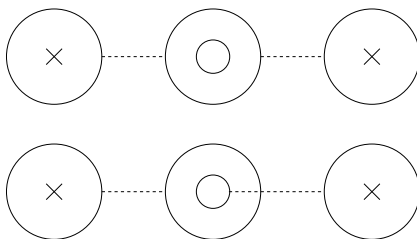
$$\frac{1}{\omega_1^2} \longrightarrow \frac{1}{\omega_1^2 - \omega_1 \vec{k}_1 \cdot (\vec{k}_1 - \vec{k}_2)/M}.$$

This shift reproduces the earlier result (2.1), which was obtained by a simple quantum mechanical derivation. This, then, shows that the formalism developed in [1] properly accounts for the contributions from off-shell D-branes<sup>◇</sup>.

### 3. Renormalization of the D-brane Tension

In order to set up the calculation of the tension renormalization, we first study the bosonic case, where such corrections are non-vanishing. The notorious singularities associated to open and closed string tachyons that plague bosonic string theory can be easily isolated and do not affect the formulation of higher order corrections to the D-brane mass.

We start by reminding the reader that the mass of the brane was obtained earlier (in equation (2.2)) by factorizing onto the translational zero-mode poles. This suggests a simple generalization to account for higher order corrections. Consider the scattering amplitude involving closed string tachyons described in equation (2.4), calculated now on the annulus. By letting the two vertex operators approach either boundary of the annulus, as in Fig. 3, the amplitude factorizes onto intermediate open string states.



**Fig. 3:** Factorization on the annulus.

In the limit depicted in Fig. 3, one can identify the one-loop correction to the translational zero-mode propagator

$$\frac{1}{\omega^2} \longrightarrow \frac{1}{\omega^2 + \Sigma(\omega^2)},$$

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<sup>◇</sup> From (2.5) one can also recover the contact interactions mentioned in a previous footnote.

where

$$\Sigma(\omega^2) = \frac{M_0 g^2}{64 \lambda_0^2} \left\langle \oint ds_1 e^{i\omega X_0(s_1)} \partial_n \vec{X}(s_1) \oint ds_2 e^{-i\omega X_0(s_2)} \partial_n \vec{X}(s_2) \right\rangle \quad (3.1)$$

and  $M_0$  and  $\lambda_0$  are the tree level mass and coupling, respectively. To find the mass correction to the D-brane, we need to extract the contributions to the amplitude arising from intermediate translational zero-modes, of the form  $\vec{k}_1 \cdot \vec{k}_2 / \omega^2$ , as in equation (2.2). The one-loop corrected amplitude,  $A$ , obtained in this way is  $A = A_0 + \Delta A$ , where

$$\Delta A = -\frac{\lambda_0^2}{M_0^2} \frac{\vec{k}_1 \cdot \vec{k}_2}{\omega^2} \Sigma'(0). \quad (3.2)$$

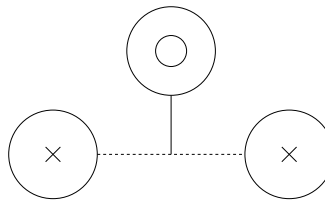
One recognizes from this expression that the mass correction,  $\Delta M$ , corresponds to the wavefunction renormalization of the translational zero-modes [5]; we find

$$\Delta M = M_0 \Sigma'(0). \quad (3.3)$$

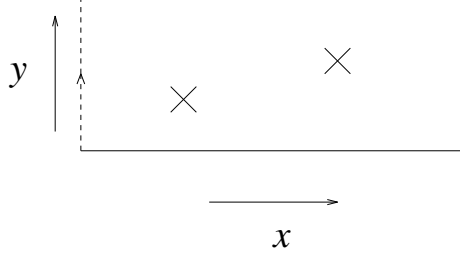
For this approach to be meaningful, there should be no double pole at  $\omega^2 = 0$ . Indeed, the presence of such a double pole would correspond to generating a mass for the translational zero-modes, which in turn implies that the D-brane mass is position dependent. The residue of this double pole is proportional to the two-point function for zero modes on the annulus

$$\Sigma(0) \propto \left\langle \oint ds_1 \partial_n \vec{X}(s_1) \oint ds_2 \partial_n \vec{X}(s_2) \right\rangle_{\text{Annulus}}, \quad (3.4)$$

where the line integrals are evaluated on the boundaries of the annulus implied in Fig. 3. The various terms cancel among themselves, leaving one term related to the existence, at this order, of an open string tachyon tadpole, depicted in Fig. 4. Such a term is not present in the supersymmetric case, so we consider it merely a symptom of bosonic string pathology.



**Fig. 4:** Contribution due to the one-loop open string tachyon tadpole.



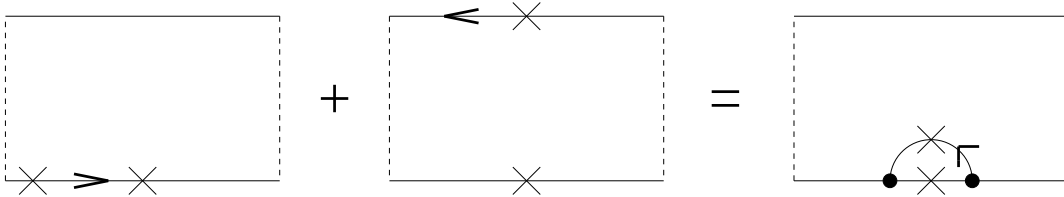
**Fig. 5:** Parameterization of the annulus by a strip.

This result, namely that the double pole receives a contribution solely from the presence of an open string tachyon tadpole, is easily obtained by using the analytic properties of the Dirichlet Green's function on the annulus. It is most convenient to parameterize the annulus as a finite strip in the upper half plane, with the opposite ends identified, as in Fig. 5.

The double pole residue calculated from (3.4) takes the explicit form

$$\Sigma(0) \propto \int \frac{da}{a^3} \left[ \prod_{n=1}^{\infty} (1 - a^{2n}) \right]^{-24} \oint ds_1 \oint ds_2 \partial_n \partial_n G_D(s_1, s_2), \quad (3.5)$$

where  $a$  is the modulus of the annulus and the integrals are over both boundaries. Using Gauss' theorem and introducing a cutoff on the integrals, one can deform the contour of integration of one of the vertex operators to a semicircle of radius  $\epsilon$ , shown in Fig. 6. In this way, we isolate the divergence due to the operators becoming close and find that there is a  $1/\epsilon$  cutoff dependence. This corresponds to the open string tachyon tadpole, as indicated in Fig. 4.



**Fig. 6:** The deformation of contours leading to the tachyon tadpole.

We next turn to the evaluation of the mass correction by calculating the residue of the single pole in  $\omega^2$ ,  $\Sigma'(0)$ . Equations (3.1) and (3.3) enable us to identify the mass correction

$$\Delta M = \frac{g^2 M_0^2}{64 \lambda_0^2} \frac{\partial}{\partial(\omega^2)} \Big|_{\omega=0} \left\langle \oint ds_1 e^{i\omega X_0(s_1)} \partial_n \vec{X}(s_1) \oint ds_2 e^{-i\omega X_0(s_2)} \partial_n \vec{X}(s_2) \right\rangle_{\text{Annulus}}. \quad (3.6)$$

Disregarding contributions which are proportional to open string tachyon tadpoles, we obtain

$$\Delta M = -\frac{1}{2\pi} \int_0^1 \frac{da}{a^3} \left[ \prod_{n=1}^{\infty} (1 - a^{2n}) \right]^{-24}, \quad (3.7)$$

which is simply the one-loop free energy.

We are now in a position to apply this approach to calculating mass corrections in the supersymmetric case. For the sake of clarity, we focus on the 0-branes of Type IIA string theory. Our approach can be easily generalized to higher  $p$ -branes by suitably compactifying the appropriate number of Neumann directions, thereby making the mass of the brane finite. In what follows, we show that the mass corrections vanish as one sums over different spin structures. This result is expected, since D-branes are BPS saturated states, and should be considered as a warm-up for calculating finite corrections to physical quantities involving non-BPS saturated states.

To calculate mass corrections in the supersymmetric case, we consider the supersymmetric extension of (3.1)

$$\begin{aligned} \Sigma(\omega^2) \propto & \left\langle \oint ds_1 e^{i\omega X_0(s_1)} \left( \partial_n \vec{X}(s_1) + i\omega \psi_0(s_1) \vec{\psi}(s_1) \right) \right. \\ & \times \left. \oint ds_2 e^{-i\omega X_0(s_2)} \left( \partial_n \vec{X}(s_2) - i\omega \psi_0(s_2) \vec{\psi}(s_2) \right) \right\rangle. \end{aligned} \quad (3.8)$$

The addition over spin structures factors out of the calculation for the correlation functions involving worldsheet bosons

$$\left\langle \oint ds_1 e^{i\omega X_0(s_1)} \partial_n \vec{X}(s_1) \oint ds_2 e^{-i\omega X_0(s_2)} \partial_n \vec{X}(s_2) \right\rangle.$$

As is well known, the sum over spin structures vanishes by the abstruse identity. Therefore both the single and double pole residues receive no contribution from worldsheet bosons, thus

$$\Sigma'(0) \propto \left\langle \oint ds_1 \psi_0(s_1) \vec{\psi}(s_1) \oint ds_2 \psi_0(s_2) \vec{\psi}(s_2) \right\rangle \quad (3.9)$$

and  $\Sigma(0) = 0$ . To further evaluate  $\Sigma'(0)$ , note that contributions from worldsheet fermion correlation functions are periodic regardless of the spin structure. Therefore, one can again analytically deform the integration contour of one of the vertex operators, as in Fig. 6. This once again only results in a contribution which is proportional to a  $1/\epsilon$  cutoff dependence. In this case, since the result is the same for all spin structures, it vanishes by the abstruse identity.



## 4. Conclusions

D-branes are an important ingredient in the various dualities recently discovered in string theory. In the weak coupling limit, these solitons are heavy and can therefore be described reasonably well by classical mechanics. We have shown how, in the first quantized approach to string theory (*i.e.*, the sum over worldsheets), off-shell D-branes are included and contribute to scattering matrix elements. This is a preliminary step in developing the quantum mechanics of D-branes. In order to obtain these results, it was crucial to introduce collective coordinates for the D-brane.

Collective coordinates, needed for the study of the dynamics of D-branes, also provide a means of calculating higher order corrections to the brane's tension. For the superstring, this allowed us to recover, using a worldsheet approach, the well known non-renormalization of the D-brane tension. One can envisage other physical quantities to which higher order corrections are nonvanishing. It should be possible, for example, to calculate the corrections to the interaction potential between branes that are moving with respect to each other [6].

In summary, a better understanding of the dynamics of D-branes should shed more light on the structure of D-branes and might hopefully give some clues to the reasons for the existence of dualities.

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